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ω-COVER, k-COVER AND RELATED SPACES ON THE PIXLEY-ROY HYPERSPACE $\mathrm{PR}[X]$

Nguyen Xuan Truc[1*], Nguyen Thi Hong Phuc[1], Ong Van Tuyen[2], and Luong Quoc Tuyen[1]

[1]*Department of Mathematics, The University of Da Nang, Unversity of Science and Education, Vietnam*

[2]*Hoa Vang High School, Da Nang City, Vietnam*

[*]*Corresponding author, Email: nxtruc2003@gmail.com*

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Abstract

In this paper, we study the concepts of ω-cover, k-cover and certain spaces defined by them on the hyperspace $\mathrm{PR}[X]$ of finite subsets of a space X endowed with the Pixley-Roy topology. We prove that $\mathrm{PR}[X]$ is an ω-Rothberger (resp., ω-Menger, ω-Lindelöf) space if and only if X is countable. Moreover, we show that $\mathrm{PR}[X]$ is a k-Lindelöf and first-countable space if and only if X is a countable and first-countable space.

Keywords: *Pixley-Roy hyperspace, ω-cover, k-cover.*

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ω-PHỦ, k-PHỦ VÀ CÁC KHÔNG GIAN LIÊN QUAN TRÊN SIÊU KHÔNG GIAN PIXLEY-ROY $\mathrm{PR}[X]$

Nguyễn Xuân Trúc[1*], Nguyễn Thị Hồng Phúc[1], Ông Văn Tuyên[2] và Lương Quốc Tuyển[1]

[1]*Khoa Toán, Trường Đại học Sư Phạm, Đại học Đà Nẵng, Việt Nam*

[2]*Trường THPT Hòa Vang, Thành phố Đà Nẵng, Việt Nam*

[*]*Tác giả liên hệ: Nguyễn Xuân Trúc, Email: nxtruc2003@gmail.com*

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Tóm tắt

Trong bài báo này, chúng tôi nghiên cứu các khái niệm về ω-phủ, k-phủ và các không gian được định nghĩa bởi chúng trên siêu không gian $\mathrm{PR}[X]$ gồm các tập hữu hạn của một không gian X với topo Pixley-Roy. Chúng tôi chứng minh rằng $\mathrm{PR}[X]$ là không gian ω-Rothberger (tương ứng, ω-Menger, ω-Lindelöf) khi và chỉ khi X là tập đếm được. Ngoài ra, chúng tôi chỉ ra rằng $\mathrm{PR}[X]$ là không gian k-Lindelöf và thỏa mãn tiên đề đếm được thứ nhất khi và chỉ khi X là không gian đếm được và thỏa mãn tiên đề đếm được thứ nhất.

Từ khóa: *Siêu không gian Pixley-Roy, ω-phủ, k-phủ.*

1. Introduction

Recently, the generalized metric properties on hyperspaces with the Pixley-Roy topology have been studied by many authors (Li, 2023; Huynh et al., 2023; Kočinac et al., 2022, Luong & Ong, 2023; Luong & Ong, 2024). The authors studied some concepts cover and related spaces on Pixley-Roy hyperpspaces such as starcompact, stric $\mathfrak{B}_0$-space, P-space, quasi-Rothberger, quasi-Lindelöf, quasi-Menger, quasi-Hurewicz and Hurewicz separability. Li (2023) reported that $\mathrm{PR}[X]$ is quasi-Rothberger (resp., quasi-Menger) if and only if X satisfies $S_1(\Pi_{rfc-h}, \Pi_{wrcf-h})$ (resp., $S_{fin}(\Pi_{rfc-h}, \Pi_{wrcf-h})$). With the aim of investigating results equivalent to properties of the Rothberger and Menger types, in this paper we study the concepts of ω-cover (resp., k-cover) and certain spaces defined by ω-covers (resp., k-covers) on the Pixley-Roy hyperspace $\mathrm{PR}[X]$.

2. Theoretical Background

Throughout this paper, $\mathbb{N}$ denotes the set of all positive integers, all spaces are assumed to be Hausdorff, other concepts and terms are understood in their usual sense unless otherwise specified (Engelking, 1989). Moreover, if $\mathcal{A}$ is a family of subsets of a topology space X, then $\bigcup\mathcal{A} = \bigcup\{A : A \in \mathcal{A}\}$.

The set $\mathrm{PR}[X]$ is the set of all non-empty finite subsets of a space X. For each $F \in \mathrm{PR}[X]$ and $A \subset X$, denote $[F, A] = \{H \in \mathrm{PR}[X] : F \subset H \subset A\}$.

The Pixley-Roy hyperspace $\mathrm{PR}[X]$ over a space X, defined by C. Pixley and P. Roy (Pixley & Roy, 1969), with the topology generated by the sets of the form $[F, V]$, where $F \in \mathrm{PR}[X]$ and V is an open subset in X containing F. For any space X, $\mathrm{PR}[X]$ is zero-dimensional, completely regular and hereditarily metacompact (Douwen, 1977).

For each $n \in \mathbb{N}$, let $\mathrm{PR}_n[X] = \{F \in \mathrm{PR}[X] : |F| \le n\}$. Then, $\mathrm{PR}[X] = \bigcup_{n=1}^{\infty}\mathrm{PR}_n[X]$ and $\mathrm{PR}_n[X] \subset \mathrm{PR}_{n+1}[X]$ for each $n \in \mathbb{N}$.

Remark 2.1. Let X be a space and $n \in \mathbb{N}$.

(1) $\mathrm{PR}_n[X]$ is a closed subspace of $\mathrm{PR}[X]$ and in particular, $\mathrm{PR}_1[X]$ is a closed discrete subspace of $\mathrm{PR}[X]$ (Tanaka, 1983).

(2) Every $\mathrm{PR}_m[X]$ is a closed subspace of $\mathrm{PR}_n[X]$ for each $m, n \in \mathbb{N}$, $m < n$ (Kočinac et al., 2022).

Definition 2.2 (Caruvana et al., 2024)**.** Let X be a space and $\mathcal{U}$ be an open cover of X. Then, $\mathcal{U}$ is said to be

(1) an ω*-cover* of X if every finite subset of X is contained in a member of $\mathcal{U}$.
(2) a k*-cover* of X if every compact subset of X is contained in a member of $\mathcal{U}$.

Definition 2.3 (Caruvana et al., 2024)**.** Let X be a space. Then, X is said to be

(1) ω*-Lindelöf (resp.* k*-Lindelöf)* if every ω-cover *(resp.* k*-cover)* of X has a countable subset as an ω-cover *(resp.* k*-cover)* of X.

(2) *ω-Menger (resp. k-Menger)* if for every $\{\mathcal{U}_n : n \in \mathbb{N}\}$ is a sequence of ω-covers *(resp. k-covers)* of X, there exists a sequence $\{\mathcal{V}_n : n \in \mathbb{N}\}$ such that for each $n \in \mathbb{N}$, $\mathcal{V}_n$ is a finite subfamily of $\mathcal{U}_n$ and $\bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ is an ω-cover *(resp. k-cover)* of X.

(3) *ω-Rothberger (resp. k-Rothberger)* if for every $\{\mathcal{U}_n : n \in \mathbb{N}\}$ is a sequence of ω-covers (*resp. k-covers)* of X, there exists, for each $n \in \mathbb{N}$, $U_n \in \mathcal{U}_n$ such that $\{U_n : n \in \mathbb{N}\}$ is an *ω-cover (resp. k-cover)* of X.

Definition 2.4 (Engelking, 1989)**.** Let X be a space. Then, X is said to be

(1) *first-countable* if for every point x of X, there exists a countable base.

(2) *second-countable* if X has a countable base.

Lemma 2.5 (Kočinac et al., 2022, Lemma 2)**.** *Let X be a space. If $\mathcal{C}$ is a compact subset of* $\mathrm{PR}[X]$, *then $\bigcup\mathcal{C}$ is a compact subset of X.*

Lemma 2.6 (Huynh et al., 2022, Theorem 3.1.1)**.** *Let X be a space. Then, X is a first-countable space if and only if so is* $\mathrm{PR}[X]$.

3. Main Results

Lemma 3.1. *Let $\mathcal{U}$ be a k-cover of a space X. Then,* $\mathfrak{U} = \left\{ \bigcup_{x\in U} [\{x\}, U] : U \in \mathcal{U} \right\}$ *is a k-cover of* $\mathrm{PR}[X]$.

Proof. Suppose that $\mathcal{U}$ is a k-cover of X and $\mathcal{A}$ is a compact subset of $\mathrm{PR}[X]$. Then, $\bigcup\mathcal{A}$ is a compact subset of X by Lemma 2.5. Because $\mathcal{U}$ is a k-cover of X, there exists $U_{\mathcal{A}} \in \mathcal{U}$ such that $\bigcup\mathcal{A} \subset U_{\mathcal{A}}$. This implies that for each $A \in \mathcal{A}$, if we pick $a \in A \subset \bigcup\mathcal{A} \subset U_{\mathcal{A}}$, then $A \in [\{a\}, U_{\mathcal{A}}]$. It shows that $\mathcal{A} \subset \bigcup_{x\in U_{\mathcal{A}}} [\{x\}, U_{\mathcal{A}}]$. Hence, $\mathfrak{U}$ is a k-cover of $\mathrm{PR}[X]$.

Lemma 3.2. *Let X be a space. Then,* $\left\{ \bigcup_{x\in F} [\{x\}, X] : F \in \mathrm{PR}[X] \right\}$ *is an ω-cover of* $\mathrm{PR}[X]$.

Proof. Suppose that $\mathcal{A}$ is a finite subset of $\mathrm{PR}[X]$. Then, $\bigcup\mathcal{A} \in \mathrm{PR}[X]$. Moreover, for each $A \in \mathcal{A}$, if we take $a \in A \subset \bigcup\mathcal{A}$, then $A \in [\{a\}, X] \subset \bigcup_{x\in\bigcup\mathcal{A}} [\{x\}, X]$.

It shows that $\mathcal{A} \subset \bigcup_{x\in\bigcup\mathcal{A}} [\{x\}, X]$.

Hence, the proof is completed.

Theorem 3.3. *The following statements are equivalent for a space X*:

(1) $\mathrm{PR}[X]$ *is an ω-Rothberger space;*

(2) $\mathrm{PR}[X]$ is an ω-Menger space;
(3) $\mathrm{PR}[X]$ is an ω-Lindelöf space;
(4) X is countable.

Proof. $(1)\Rightarrow(2)$. Let $\mathrm{PR}[X]$ be an ω-Rothberger space and $\{\mathfrak{U}_n : n\in\mathbb{N}\}$ be a sequence of ω-covers of $\mathrm{PR}[X]$. Then, for each $n\in\mathbb{N}$, there exists $\mathcal{U}_n \in \mathfrak{U}_n$ such that $\{\mathcal{U}_n : n\in\mathbb{N}\}$ is an ω-cover of $\mathrm{PR}[X]$. Now, for each $n\in\mathbb{N}$, if we put $\mathfrak{V}_n = \{\mathcal{U}_n\}$, then $\mathfrak{V}_n$ is a finite subfamily of $\mathfrak{U}_n$ and $\bigcup_{n\in\mathbb{N}} \mathfrak{V}_n = \{\mathcal{U}_n : n\in\mathbb{N}\}$ is an ω-cover of $\mathrm{PR}[X]$. Hence, $\mathrm{PR}[X]$ is an ω-Menger space.

$(2)\Rightarrow(3)$. Let $\mathrm{PR}[X]$ be an ω-Menger space and $\mathfrak{U}$ be an ω-covers of $\mathrm{PR}[X]$. For each $n\in\mathbb{N}$, we put $\mathfrak{V}_n = \mathfrak{U}$. Then, $\{\mathfrak{V}_n : n\in\mathbb{N}\}$ is a sequence of ω-covers of $\mathrm{PR}[X]$. Since $\mathrm{PR}[X]$ is an ω-Menger space, for each $n\in\mathbb{N}$, there exists $\mathfrak{W}_n$ is a finite subfamily of $\mathfrak{V}_n$ such that $\bigcup_{n\in\mathbb{N}} \mathfrak{W}_n$ is an ω-cover of $\mathrm{PR}[X]$. This implies that $\bigcup_{n\in\mathbb{N}} \mathfrak{W}_n$ is a countable ω-subcover of $\mathfrak{U}$. Hence, $\mathrm{PR}[X]$ is an ω-Lindelöf space.

$(3)\Rightarrow(4)$. Assume that $\mathrm{PR}[X]$ is an ω-Lindelöf space. By Lemma 3.2,

$$\left\{\bigcup_{x\in F}\left[\{x\}, X\right] : F\in \mathrm{PR}[X]\right\}$$

is an ω-cover of $\mathrm{PR}[X]$. Because $\mathrm{PR}[X]$ is an ω-Lindelöf space, there exists $\{F_m : m\in\mathbb{N}\}$ is a sequence of members of $\mathrm{PR}[X]$ such that $\left\{\bigcup_{x\in F_m}\left[\{x\}, X\right] : m\in\mathbb{N}\right\}$ is an ω-cover of X. Now, we will prove that X is countable. Let $y\in X$. Then, $\{y\}\in \mathrm{PR}[X]$. This implies that there exists $m_y\in\mathbb{N}$ such that $\{y\}\in \bigcup_{x\in F_{m_y}}\left[\{x\}, X\right]$. It shows that there exists $x_y\in F_{m_y}$ such that $\{y\}\in\left[\{x_y\}, X\right]$. Thus, $y = x_y$. Therefore, we claim that $X = \bigcup_{y\in X}\{y\} = \bigcup_{y\in X}\{x_y\} \subset \bigcup_{y\in X} F_{m_y} \subset \bigcup_{m\in\mathbb{N}} F_m$.

Hence, X is countable.

$(4)\Rightarrow(1)$. Let $\{\mathfrak{U}_n : n\in\mathbb{N}\}$ be a sequence of ω-covers of $\mathrm{PR}[X]$. Because X is countable, $\mathrm{PR}[X]$ is countable. Now, we put $\{\mathcal{A}\subset \mathrm{PR}[X] : \mathcal{A} \text{ is finite}\} = \{\mathcal{F}_m : m\in\mathbb{N}\}$.

For each $m\in\mathbb{N}$, since $\mathfrak{U}_m$ is an ω-cover of X, there exists $\mathcal{U}_m\in\mathfrak{U}_m$ such that $\mathcal{F}_m\subset\mathcal{U}_m$. This implies that $\{\mathcal{U}_m : m\in\mathbb{N}\}$ is an ω-cover of $\mathrm{PR}[X]$. Hence, $\mathrm{PR}[X]$ is an ω-Rothberger space.

Theorem 3.4. *Let X be a space. Then, $\mathrm{PR}[X]$ is a k-Lindelöf and first-countable space if and only if X is a countable and first-countable space.*

Proof. Necessity. Assume that $\mathtt{PR}[X]$ is a k-Lindelöf and first-countable space. By Lemma 2.6, X is a first-countable space. Next, let $\mathcal{K}$ be an arbitrary compact subset of $\mathtt{PR}[X]$. Then, $\{[F,X]: F \in \mathcal{K}\}$ is an open cover of $\mathcal{K}$ in $\mathtt{PR}[X]$. This implies that there exists $\left\{F_{\mathcal{K}}^{(1)},\dots,F_{\mathcal{K}}^{(m_{\mathcal{K}})}\right\} \subset \mathcal{K}$ such that $\mathcal{K} \subset \bigcup_{i=1}^{m_{\mathcal{K}}}\left[F_{\mathcal{K}}^{(i)}, X\right]$.

Therefore, we claim that $\left\{\bigcup_{i=1}^{m_{\mathcal{K}}}\left[F_{\mathcal{K}}^{(i)}, X\right] : \mathcal{K} \subset \mathtt{PR}[X]\right\}$ is a k-cover of $\mathtt{PR}[X]$. Because $\mathtt{PR}[X]$ is a k-Lindelöf space, there exists $\{\mathcal{K}_s : s \in \mathbb{N}\}$ is a sequence of compact subsets of $\mathtt{PR}[X]$ such that $\left\{\bigcup_{i=1}^{m_{\mathcal{K}_s}}\left[F_{\mathcal{K}_s}^{(i)}, X\right] : s \in \mathbb{N}\right\}$ is a k-cover of $\mathtt{PR}[X]$. Now, we will prove that X is countable. In fact, let $x \in X$. Then, $\{x\} \in \mathtt{PR}[X]$. This implies that there exists $s_x \in \mathbb{N}$ such that $\{x\} \in \bigcup_{i=1}^{m_{\mathcal{K}_{s_x}}}\left[F_{\mathcal{K}_{s_x}}^{(i)}, X\right]$.

It shows that there exists $i_x \le m_{\mathcal{K}_{s_x}}$ such that $\{x\} \in \left[F_{\mathcal{K}_{s_x}}^{(i_x)}, X\right]$. Thus, $\{x\} = F_{\mathcal{K}_{s_x}}^{(i_x)}$. Therefore,

$$X = \bigcup_{x\in X}\{x\} = \bigcup_{x\in X} F_{\mathcal{K}_{s_x}}^{(i_x)} \subset \bigcup_{x\in X}\bigcup_{i=1}^{m_{\mathcal{K}_{s_x}}} F_{\mathcal{K}_{s_x}}^{(i)} \subset \bigcup_{s\in\mathbb{N}}\bigcup_{i=1}^{m_{\mathcal{K}_s}} F_{\mathcal{K}_s}^{(i)}.$$

Hence, X is countable.

Sufficiency. Suppose that X is a countable and first-countable space. By Lemma 2.6, $\mathtt{PR}[X]$ is a first-countable space. Since X is countable, $\mathtt{PR}[X]$ is also countable. This implies that $\mathtt{PR}[X]$ has a countable base $\mathfrak{B}$. We put $\mathfrak{A} = \left\{\bigcup\mathfrak{C} : \mathfrak{C} \subset \mathfrak{B}, \mathfrak{C} \text{ is finite}\right\}$.

It easy to see that $\mathfrak{A}$ is countable. Let $\mathfrak{U}$ be an arbitrary k-cover of $\mathtt{PR}[X]$ and $\mathcal{K}$ be an arbitrary compact set in $\mathtt{PR}[X]$. Then, there exists $\mathcal{U}_{\mathcal{K}} \in \mathfrak{U}$ such that $\mathcal{K} \subset \mathcal{U}_{\mathcal{K}}$. Because $\mathcal{U}_{\mathcal{K}}$ is an open subset of $\mathtt{PR}[X]$, there exists a subfamily $\mathfrak{D}_{\mathcal{U}_{\mathcal{K}}}$ of $\mathfrak{B}$ such that $\mathcal{U}_{\mathcal{K}} = \bigcup\mathfrak{D}_{\mathcal{U}_{\mathcal{K}}}$. This implies that $\mathfrak{D}_{\mathcal{U}_{\mathcal{K}}}$ is an open cover of $\mathcal{K}$ in $\mathtt{PR}[X]$. Therefore, there exists a finite subfamily $\mathfrak{D}'_{\mathcal{U}_{\mathcal{K}}}$ of $\mathfrak{D}_{\mathcal{U}_{\mathcal{K}}}$ such that $\mathcal{K} \subset \bigcup\mathfrak{D}'_{\mathcal{U}_{\mathcal{K}}} \subset \mathcal{U}_{\mathcal{K}}$.

Now, if we put $\mathfrak{F} = \left\{\bigcup\mathfrak{D}'_{\mathcal{U}_{\mathcal{K}}} : \mathcal{K} \subset \mathtt{PR}[X], \mathcal{K} \text{ is compact}\right\}$, then $\mathfrak{F}$ is a subfamily of $\mathfrak{A}$. Thus, $\mathfrak{F}$ is countable. For each $\mathcal{V} \in \mathfrak{F}$, we pick $\mathcal{U}_{\mathcal{V}} \in \mathfrak{U}$ such that $\mathcal{V} \subset \mathcal{U}_{\mathcal{V}}$. Then, $\{\mathcal{U}_{\mathcal{V}} : \mathcal{V} \in \mathfrak{F}\}$ is a countable k-cover of $\mathtt{PR}[X]$. Hence, $\mathtt{PR}[X]$ is a k-Lindelöf space.

Example 3.5. *An example of a space X that is ω-Rothberber (resp., ω-Menger, ω-Lindelöf)*

Solution. Let X be a countable set with an arbitrary topology.

Claim 1. X is an ω-Rothberger (resp. ω-Menger) space.

Assume that $\{\mathcal{U}_m : m \in \mathbb{N}\}$ is a sequence of ω-covers of X. Because X is countable,

$$\{F \subset X : F \text{ is finite}\} = \{F_k : k \in \mathbb{N}\}.$$

For each $k \in \mathbb{N}$, since $\mathcal{U}_k$ is an ω-cover of X, there exists $U_k \in \mathcal{U}_k$ such that $F_k \subset U_k$.

Then, $\{U_k : k \in \mathbb{N}\}$ is an ω-cover of X. Thus, X is an ω-Rothberger space.

Now, for each $m \in \mathbb{N}$, if we put $\mathcal{V}_m = \{U_m\}$, then $\mathcal{V}_m$ is a finite subfamily of $\mathcal{U}_m$ and $\bigcup_{m \in \mathbb{N}} \mathcal{V}_m$ is an ω-cover of X. Hence, X is an ω-Menger space.

Claim 2. X is an ω- Lindelöf space.

Assume that $\mathcal{U}$ is an ω-cover of X. Because X is countable,

$$\{F \subset X : F \text{ is finite}\} = \{F_k : k \in \mathbb{N}\}.$$

Since $\mathcal{U}$ is a ω-cover of X, for each $k \in \mathbb{N}$, there exists $U_k \in \mathcal{U}$ such that $F_k \subset U_k$. Therefore, $\{U_k : k \in \mathbb{N}\}$ is an ω-cover of X. Hence, X is ω- Lindelöf space.

4. Conclusion

In this paper, we present and provide detailed proofs of some new results concerning the equivalence of ω-cover (k-cover) and certain properties defined by them between the topological space X and the Pixley-Roy hyperspace $\mathrm{PR}[X]$. These results contribute to enriching the field of research on generalized metric properties in general topology.

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