PRESENTING A SURFACE OF REVOLUTION BY USING ORTHONORMAL PROJECTION WITH THE TIKZ PACKAGE

Tran Le Nam[1*] and Phan Thi Hiep[2]

[1]*Faculty of Mathematics - Informatics Teacher Education, Dong Thap University, Vietnam*

[2]*Faculty of Primary and Pre-School Education, Dong Thap University, Vietnam*

[*]*Corresponding author: Tran Le Nam, Email: tranlenam@dthu.edu.vn*

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Abstract

Orthonormal projections emerge from subjecting 3-dimensional vectors to orthogonal transformations and projecting them to 2 dimensions. They are not to be confused with the perspective projections, which are more realistic. Orthonormal projections may be thought of a limit of perspective projections at large distances, where large means that the distance of the observer is much larger than the dimensions of the objects that get depicted. The paper will calculate the critical points of light on revolution surfaces. After that, the TikZ library in the LaTeX compiler is used to represent cylinders, cones, and their curves accurately. Thereby, readers can draw most of the figures representing the revolutions in Mathematical subject, at the high school level.

Keywords: *Orthonormal projections, revolution, figures representing, TikZ, LaTeX.*

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BIỂU DIỄN CÁC MẶT TRÒN XOAY QUA PHÉP CHIẾU TRỰC GIAO VỚI GÓI TIKZ

Trần Lê Nam[1*] và Phan Thị Hiệp[2]

[1]*Khoa Sư phạm Toán - Tin, Trường Đại học Đồng Tháp, Việt Nam*

[2]*Khoa Giáo dục Tiểu học - Mầm non, Trường Đại học Đồng Tháp, Việt Nam*

[*]*Tác giả liên hệ: tranlenam@dthu.edu.vn*

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Tóm tắt

Phép chiếu trực giao nhúng các véc-tơ 3-chiều qua phép biến đổi trực giao và chiếu chúng vào mặt phẳng. Phép chiếu này khác với phép phối cảnh, nó mô tả các đối tượng thực tế hơn. Các phép chiếu trực giao có thể xem là giới hạn của phép phối cảnh với khoảng cách lớn, nghĩa là người quan sát lớn và rất xa đối tượng được mô tả. Bài viết sẽ tính các điểm giới hạn ánh sáng trên các mặt tròn xoay. Từ đó, chúng tôi sử dụng thư viện TikZ trong ngôn ngữ LaTeX để biểu diễn các mặt trụ, mặt nón và các đường cong nằm trên chúng một cách chính xác. Dựa vào đây, bạn đọc có thể biểu diễn hầu hình các hình liên quan đến mặt tròn xoay trong môn Toán ở bậc trung học cơ sở.

Từ khóa: *Phép chiếu trực giao, mặt tròn xoay, biểu diễn hình, TikZ, LaTeX.*

1. Introduction

The revolutions are the basic geometric objects in high schools. In grade 9^{th} geometry, students study with canonical spheres, cylinders, and cones. However, some pictures must be comminated by two or more objects together. Furthermore, some pictures are not canonical. High school students are introduced to conics, intersections of cones with planes, and curves lying on noncanonical revolutions. The representations of these objects on the plane can cause some difficulties for teachers in preparing documents. Difficulties often determine the coordinates of the critical point of light on rotating circles through orthogonal projection. We must apply the results of parameter curves and revolution in the differential geometry (Manfredo, 2016).

LaTeX is a software system for document preparation (Lamport, 1986; Pablo, 2017). When writing, the writer uses plain text as opposed to the formatted text found in “What You See Is What You Get” word processors like Microsoft Word, LibreOffice Writer, and Apple Pages. The writer uses markup tagging conventions to define the general structure of a document (such as article, book, and letter), stylize text throughout a document (such as bold and italics), and citations and cross-references. A TeX distribution such as TeX Live or MiKTeX is used to produce an output file (such as PDF or DVI) suitable for printing or digital distribution.

LaTeX is widely used in academia for the communication and publication of scientific documents in many fields, including mathematics, computer science, engineering, physics, chemistry, economics, linguistics, quantitative psychology, philosophy, and political science. It also has a prominent role in the preparation and publication of books and articles that contain complex multilingual materials, such as Sanskrit and Greek. LaTeX uses the TeX typesetting program for formatting its output, and is itself written in the TeX macro language.

Because of convenience in managing large resources, not imaged documents like the Mathtype software. More and more teachers are using LaTeX software to compile mathematical materials in Vietnam. They used to draw the picture by The PGF/TikZ package, a pair of languages for producing vector graphics (e.g., technical illustrations and drawings) from a geometric/ algebraic description, with standard features including the drawing of points, lines, arrows, paths, circles, ellipses and polygons (Andrew & William, 2007). PGF is a lower-level language, while TikZ is a set of higher-level macros that use PGF. The top-level PGF and TikZ commands are invoked as TeX macros, but in contrast with PSTricks, the PGF/TikZ graphics themselves are described in a language that resembles MetaPost. Till Tantau is the designer of the PGF and TikZ languages. He is also the main developer of the only known interpreter for PGF and TikZ written in TeX. PGF is an acronym for “Portable Graphics Format”. TikZ was introduced in version 0.95 of PGF, and it is a recursive acronym for “TikZ ist kein Zeichenprogramm” (Claudio, 2007).

In this paper, we establish and solve the equations to determine the coordinates of the critical point of curves for the revolution surfaces, considering some cases of irregular revolution surfaces. From there we give the code to draw the rotated faces and intersecting curves on revolution surfaces by TikZ package.

2. Main contents

2.1. Orthonormal projections

The 3d-like pictures emerge by rotating the view. The conventions for the parametrization of the orthogonal rotations in terms of three rotation angles ϕ, ψ and θ are

$$O(\phi,\psi,\theta) = R_x(\theta)\cdot R_y(\psi)\cdot R_z(\phi), \text{ where}$$

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix},$$

$$R_y(\psi) = \begin{pmatrix} \cos(\psi) & 0 & -\sin(\psi) \\ 0 & 1 & 0 \\ \sin(\psi) & 0 & \cos(\psi) \end{pmatrix},$$

$$R_z(\phi) = \begin{pmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

So the matrix of $O(\phi,\psi,\theta)$ is

$$O(\phi,\psi,\theta) = \begin{pmatrix} c_\phi c_\psi & s_\phi c_\psi & -s_\psi \\ c_\phi s_\psi s_\theta - s_\phi c_\theta & s_\phi s_\psi s_\theta + c_\phi c_\theta & c_\psi s_\theta \\ c_\phi s_\psi c_\theta + s_\phi s_\theta & s_\phi s_\psi c_\theta - c_\phi s_\theta & c_\psi c_\theta \end{pmatrix}. \quad (1)$$

Here, $c_\phi := \cos\phi$, $s_\phi := \sin\phi$ and so on.

Partically, $O(\phi,0,\theta) = R_x(\theta)\cdot R_z(\phi)$ is

$$O(\phi,0,\theta) = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi\cos\theta & \cos\phi\cos\theta & \sin\theta \\ \sin\phi\sin\theta & -\cos\phi\sin\theta & \cos\theta \end{pmatrix}.$$

2.2. The hidden points on a cylinder

a) The cylinder of revolution about z-axis

To draw a 3d-like cylinder of revolution about z-axis, we must present the circles with equations $\begin{cases} x^2+y^2=r \\ z=z_0, \end{cases}$ where $r>0$ and $z_0\in\mathbb{R}$.

When drawing the circle in space, some parts are "visible", i.e. in the foreground, while other parts are "hidden", i.e. in the background. We will find the critical points for these points through the orthogonal rotations $O(\theta,0,\phi)$. The circle has a parameter form

$$c(t) = (r\cos t, r\sin t, z_0),\ t\in(0,2\pi),$$

represented on the drawing board by an ellipse with parameters.

$$\bar{c}(t) = (r(\cos\phi\cos t + \sin\phi\sin t),$$
$$r(-\sin\phi\cos\theta\cos t + \cos\phi\cos\theta\sin t)$$
$$+\sin\theta z_0).$$

Its tangent vector at $c(t)$ is

$$c'(t) = r(-\cos\phi\sin t + \sin\phi\cos t,$$
$$\sin\phi\cos\theta\sin t + \cos\phi\cos\theta\cos t).$$

The direction vector of y-axis is represented by

$$\vec{v} = (0,\sin\theta).$$

Therefore, the critical points t of visible points satisfied $-\cos\phi\sin t + \sin\phi\cos t = 0.$

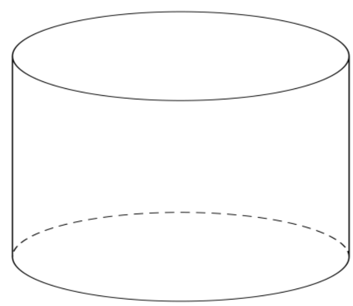
Hence, we get $\tan t = \tan\phi.$

It derives that $t=\phi$ or $t=\phi+180^\circ$.

Application these calculations, the cyclinder of revolution about z-axis can be drawn with the **tikz-3dplot** package by the following code.

https://www.overleaf.com/read/gbspcqdynwkk

```
\begin{tikzpicture}[declarefunction={r=1.9;d=4;
theta=65;phi=30; tt=phi;tth=180+tt;}]
\tdplotsetmaincoords{theta}{phi}
\begin{scope}[tdplot_main_coords]
\draw[densely dashed]
    plot[domain=tt:tth,samples=200]
    ({r*cos(\x)},{r*sin(\x)},0);
\draw plot[domain=tth:tt+360,samples=200]
    ({r*cos(\x)},{r*sin(\x)},0);
\draw plot[domain=0:360,samples=200]
    ({r*cos(\x)},{r*sin(\x)},d);
\draw ({r*cos(tt)},{r*sin(tt)},0)—
    ({r*cos(tt)},{r*sin(tt)},d)
    ({r*cos(tth)},{r*sin(tth)},0)—
    ({r*cos(tth)},{r*sin(tth)},d);
\end{scope}
\end{tikzpicture}
```



b) The intersection curve of cylinder and plane

For a general cylinder and plane, we have to calculate the parameter of the intersection curve of cylinder and plane for visible points of the curve.

Draw a simulation of the intersection of the cylinder is $(T): x^2+y^2=r^2$ and the plane is $(P): Ax+By+Cz+D=0$, with $C\neq 0$.

The parameterization of the cylinder (T) has the form

$$X(u,v) = (r\cos v, r\sin v, u),$$
$$v\in(0,2\pi), u\in\mathbb{R}.$$

Therefore, the intersection of (T) and (P) is an ellipse

$$c(t) = (r\cos t, r\sin t,$$
$$\frac{-1}{C}\cdot(Ar\cos t + Br\sin t + D)).$$

This parameter is projected onto the display plane as

$$c(t)=(r(\cos\phi\cos t+\sin\phi\sin t),$$
$$r(-\sin\phi\cos\theta\cos t+\cos\phi\cos\theta\sin t)$$
$$+\sin\theta z(t)),$$
with
$$z(t)=\frac{-1}{C}\cdot(Ar\cos t+Br\sin t+D).$$

The x-coordinate of the tangent vector of $c(t)$ is $x'(t)=r(-\cos\phi\sin t+\sin\phi\cos t)$.

Hence, the visible critical points t satisfies the equation

$$\tan t=\tan\phi\Leftrightarrow t=\phi\vee t=180+\phi.$$

Therefore, the visible critical points of the curve (c) are

$$c_1(\phi)=(r\cos\phi, r\sin\phi,$$
$$\frac{-1}{C}\cdot(Ar\cos\phi+Br\sin\phi+D))$$

and

$$c_2(\phi+180)=(-r\cos\phi,-r\sin\phi,$$
$$\frac{1}{C}\cdot(Ar\cos\phi+Br\sin\phi-D)).$$

In particularly, if the plane (P) is the xy-plane, the visible critical points have been known as $(r\cos\phi, r\sin\phi, 0)$ and

$$(-r\cos\phi,-r\sin\phi,0).$$

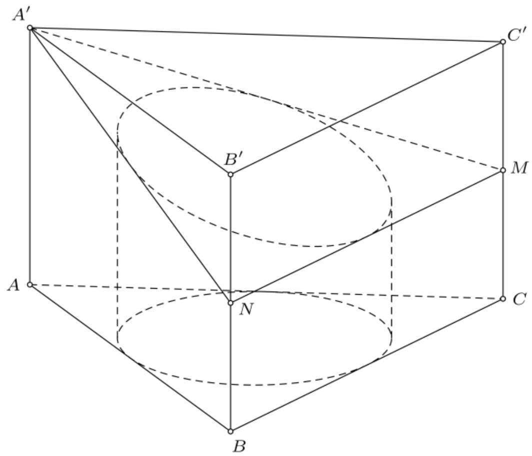
c) *An example*

This example shows the code illustrating the regular triangular prism $ABC.A'B'C'$ circumscribed to a cylinder. Especially, the intersection of the inscribed cylinder and plane $(A'MN)$ where M, N are midpoints of BB' and CC', respectively.

https://www.overleaf.com/read/bypmrfbhdmzy

```
\begin{tikzpicture}[declare function={r=4;h=2;
        rnt=r*sin(30);
        theta=70;phi=-145;
        nA=h*(sin(240)-sin(120));
        nB=h*(-cos(240)+cos(120));
        nC=r*(-sin(240)+2*sin(120));
        tt=phi;tth=180+tt;}]
 \tdplotsetmaincoords{theta}{phi}
 \begin{scope}[tdplot_main_coords]
 \path (r,0,0) coordinate (A)
 ({r*cos(120)},{r*sin(120)},0) coordinate (B)
 ({r*cos(240)},{r*sin(240)},0) coordinate (C)
 (r,0,{2*h}) coordinate (Ap)
 ({r*cos(120)},{r*sin(120)},{2*h})
coordinate (Bp)
 ({r*cos(240)},{r*sin(240)},{2*h})
coordinate (Cp)
 ($ (C)!0.5!(Cp) $) coordinate (M)
($ (B)!0.5!(Bp) $) coordinate (N);
 \draw (A)--(B)--(C) (Ap)--(A) (B)--(Bp)
 (C)--(Cp) (Ap)--(Bp)--(Cp)--cycle
 (Ap)--(N)--(M);
 \draw[densely dashed] plot [domain=0:360,
   samples=200]({rnt*cos(\x)},{rnt*sin(\x)},0)
(A)--(C) (Ap)--(M);
 \draw[densely dashed] plot[domain=0:360,
  samples=200]({rnt*cos(\x)},{rnt*sin(\x)}, {(nA*
  (rnt*cos(\x)-r)- nB*rnt*sin(\x))/nC+2*h});
 \draw[densely dashed]({rnt*cos(tt)},
 {rnt*sin(tt)},0) --({rnt*cos(tt)},{rnt*sin(tt)},
 {(-nA*(rnt*cos(tt)-r)-nB*rnt*sin(tt))/nC+2*h})
 ({rnt*cos(tth)},{rnt*sin(tth)},0)--({rnt*cos(tth)},
 {rnt*sin(tth)},{(-nA*(rnt*cos(tth)-r)-
nB*rnt*sin(tth))/nC+2*h});
 \end{scope}
 \foreach \t/\g in {A/180,B/-60,C/0,M/10,
        N/-20}{\draw[fill=white] (\t) circle (1pt)
        node[shift={(\g:7pt)},font=\scriptsize]
        {$ \t $};}
 \foreach \t/\g/\d in {Bp/80/B',Ap/120/A',
        Cp/30/C'}{
        \draw[fill=white] (\t) circle (1pt)
        node[shift={(\g:7pt)},
```

font=\scriptsize]{$ \d $};}
\end{tikzpicture}



2.3. The visible points and curves on a cones

2.3.1. The extreme points on a cones

Let's (c) is a circle

$$c(t) = (r\cos t, r\sin t, z_0),$$

where $r > 0, z_0 \in \mathbb{R}$ in space. By calculating $O(c(t))$, the curve (c) is represented by an ellipse with parameters

$$c(t) = (r(\cos\phi\cos t + \sin\phi\sin t), r(-\sin\phi\cos\theta\cos t + \cos\phi\cos\theta\sin t) + \sin\theta z_0),$$

on the plane $\mathbb{R}^2$.

Simplifying the parameter of (c), we get

$$c(t) = (r\cos(\phi - t), r\cos\theta\sin(t-\phi) + \sin\theta z_0).$$

Its tangent vectors are

$$c'(t) = r(\sin(\phi - t), \cos\theta\cos(t-\phi)).$$

The point $(0,0,d)$ is represented by the point with coordinates $(0, d\sin\theta)$ on the plane.

The tangent to (c) at t passes through the point $(0, d\sin\theta)$ if and only if

$$\frac{r\cos(\phi - t)}{\sin(\phi - t)} = \frac{r\cos\theta\sin(t-\phi) + \sin\theta z_0 - d\sin\theta}{\cos\theta\cos(t-\phi)}.$$

The above equation is equivalent to

$$r\cos^2(\phi - t) = -r\sin^2(\phi - t) + (z_0 - d)\tan\theta\sin(\phi - t).$$

Simplifying the above equation, we get

$$(z_0 - d)\tan\theta\sin(\phi - t) = r.$$

Thefore, the critical visible points t satisfied

$$\sin(\phi - t) = \frac{r\cot\theta}{(z_0 - d)}\cdot$$

From there, we get

$$\phi - t = \arcsin\frac{r\cot\theta}{(z_0 - d)}$$

or

$$\phi - t = \pi - \arcsin\frac{r\cot\theta}{(z_0 - d)}\cdot$$

It infers that $t = \phi - \arcsin\frac{r\cot\theta}{(z_0 - d)}$

or $t = \phi + \arcsin\frac{r\cot\theta}{(z_0 - d)} - 180^\circ$.

Particularly, when $z_0 = 0$, i.e the circle (c) lies on the plane (Oxy), we get

$$t = \phi + \arcsin\frac{r\cot\theta}{d}$$

$$\text{or } t = \phi + 180 - \arcsin\frac{r\cot\theta}{d}\cdot$$

2.3.2. The intersection of the cone with the plane $x = a$ and a conical frustum

The parameterization of the cone (C) is form

$$X(u,v) = (r(1-u)\cos v, r(1-u)\sin v, hu), \quad u \in (0,1), v \in (0, 2\pi), r > 0.$$

When $x = a$, with $a \in (-r, r)$, we have

$$r(1-u) = \frac{a}{\cos v}, u = \frac{r\cos v - a}{r\cos v}\cdot$$

It derives the intersection with a parameter of the form

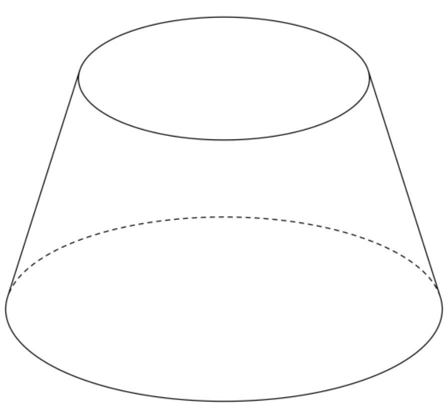
$$c(t) = \left(a, a\tan t, h\cdot\frac{r\cos v - a}{r\cos v}\right),$$

$$t \in \left(-\arccos\frac{a}{r}, \arccos\frac{a}{r}\right).$$

Applicating above result, we can draw the a conical frustum created by slicing the top off a cone (with the cut made $x = a$).

https://www.overleaf.com/read/xjxcxbjxsrdb

```
\begin{tikzpicture}
 \tikzset{declare function={r=3;rn=2;h=3.5;
 theta=65;d=(h*r)/(r-rn);tt=asin(r*cot(theta)/d);
 tth=180-asin(r*cot(theta)/d);},
samples=200,smooth}
 \tdplotsetmaincoords{theta}{0}
 \begin{scope}[tdplot_main_coords]
  \draw[dash pattern=on 2pt off 1.5pt]
      plot[domain=tt:tth]({r*cos(\x)},{r*sin(\x)},0)
         coordinate (A);
  \draw plot[domain=tth:tt+360]
       ({r*cos(\x)},{r*sin(\x)},0) coordinate (B)
      plot[domain=tth:tt+360]
      ({rn*cos(\x)},{rn*sin(\x)},h) coordinate (Bt)
      plot[domain=tt:tth]
      ({rn*cos(\x)},{rn*sin(\x)},h) coordinate (At);
  \end{scope}
  \draw (A)--(At) (B)--(Bt);
\end{tikzpicture}
```



2.3.3. The intersection of the cone with the plane and conic curves

We have already known that the intersection of a cone and plane is a conic section. Specially, we have three cases.

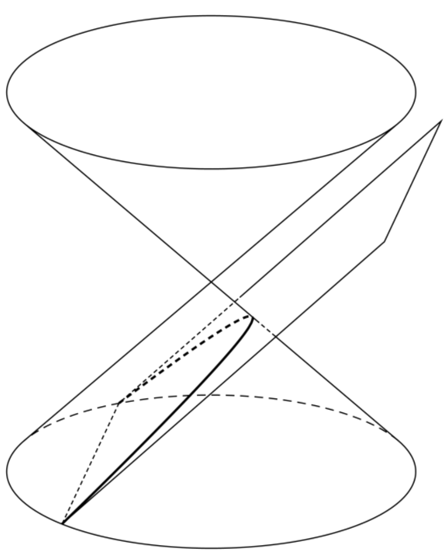
- If the plane is parallel to the base (perpendicular to the axis) of the cone, the cross section is a circle;
- If the plane is at an inclination relative to the base that is strictly between 0° and the slant angle of the cone, the cross section is an ellipse;
- If the plane is parallel to a slant height, the cross section is a parabola;
- If the plane is at an inclination that is strictly greater than the slant angle of the cone and smaller than or equal to 90°, the cross section is a hyperbola.

The below code illustrates the intersections of an ellipse and a parabola.

https://www.overleaf.com/read/dxcdxrckxrtn

```
\begin{tikzpicture}[declare function={r=2.5;
  d=r;dh=-r;theta=68;phi=10;
  tt=phi+asin(r*cot(theta)/d);
  tth=180+phi-asin(r*cot(theta)/d);
  ttd=phi+asin(r*cot(theta)/dh);
  tthd=180+phi-asin(r*cot(theta)/dh);
  ght=sqrt(r^2-1);ghtam=-sqrt(r^2-1);}]
 \tdplotsetmaincoords{theta}{phi}
 \begin{scope}[tdplot_main_coords]
 \draw[densely dashed]
plot[domain=tt:tth,samples=200]
({r*cos(\x)},{r*sin(\x)},-r) ;
 \draw plot[domain=tth:tt+360,samples=200]
({r*cos(\x)},{r*sin(\x)},-r)
 plot[domain=0:360,samples=200]
({r*cos(\x)},{r*sin(\x)},r)
   ({r*cos(ttd)},{r*sin(ttd)},r) coordinate (A)
     --(0,0,0) coordinate (O)
```

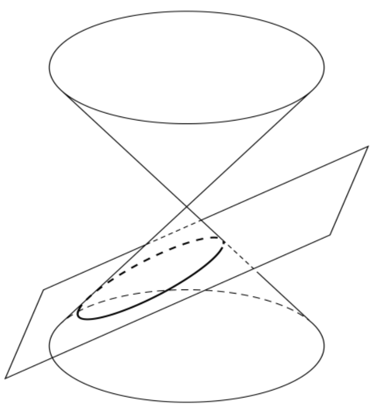
```
  --({r*cos(tthd)},{r*sin(tthd)},r) coordinate (B);
 \path ($ (O)-(A) $) coordinate (Ad)
($ (O)-(B) $) coordinate (Bd);
 \draw (O)--(Ad);
 \draw[thick,dash pattern=on 2pt off 1.5pt]
  plot[domain=0.5:{-r+1},samples=200,smooth]
   (\x,{sqrt(1-2*\x)},{\x-1})  coordinate (Ph);
 \draw[thick] plot[domain=0.5:{-r+1},
   samples=200,smooth]
   (\x,{-sqrt(1-2*\x)},{\x-1})  coordinate (Pm);
 \path ($ (Pm)+(4,0,4) $) coordinate (Pb)
          ($ (Ph)+(4,0,4) $) coordinate (Pbon)
(barycentric cs:Ph=0.625,Pbon=0.375)
coordinate (PbonN)
          (intersection of Bd--O and Ph--Pb)
 coordinate (Bdm)
          (intersection of Bd--O and Pb--Pm)
 coordinate (Bdh);
 \draw (Pm)--(Pb)--(Pbon)--(PbonN)
(O)--(Bdm) (Bdh)--(Bd);
 \draw[dashed,dash pattern=on 1.5pt off 1pt]
(Pm)--(Ph)--(PbonN) (Bdm)--(Bdh);
\end{scope}
\end{tikzpicture}
```



https://www.overleaf.com/read/xqqvpbrfsktf

```
\begin{tikzpicture}[declare function={r=2.5;d=r;
dh=-r; theta=68;phi=10;
tt=phi+asin(r*cot(theta)/d);
tth=180+phi-asin(r*cot(theta)/d);
ttd=phi+asin(r*cot(theta)/dh);
tthd=180+phi-asin(r*cot(theta)/dh);
ght=sqrt(r^2-1);ghtam=-sqrt(r^2-1);}]
 \tdplotsetmaincoords{theta}{phi}
\begin{scope}[tdplot_main_coords]
 \draw[densely dashed]
  plot[domain=tt:tth,samples=200]
  ({r*cos(\x)},{r*sin(\x)},-r) ;
 \draw plot[domain=tth:tt+360,samples=200]
  ({r*cos(\x)},{r*sin(\x)},-r)
  plot[domain=0:360,samples=200]
   ({r*cos(\x)},{r*sin(\x)},r)
   ({r*cos(ttd)},{r*sin(ttd)},r) coordinate (A)
  --(0,0,0) coordinate (O)
  --({r*cos(tthd)},{r*sin(tthd)},r) coordinate (B);
 \path ($ (O)-(A) $) coordinate (Ad)
   ($ (O)-(B) $) coordinate (Bd);
  \draw (O)--(Ad);
 \draw[thick,dashed] plot[domain=-1:0.333,
   samples=200,smooth]
   (2*\x,{sqrt(1-2*\x-3*\x*\x)},{\x-1});
 \draw[thick] plot[domain=-1:0.3333,
    samples=200,smooth]
      (2*\x,{-sqrt(1-2*\x-3*\x*\x)},{\x-1}) ;
  \path (-3,-2,-2.5) coordinate (Pm)
  (-3,2,-2.5) coordinate (Ph)
   ($ (Pm)+(6,0,3) $) coordinate (Pb)
   ($ (Ph)+(6,0,3) $) coordinate (Pbon)
   (intersection of Ph--Pbon and A--Ad)
```

```
   coordinate (PbonM)
  (intersection of Ph--Pbon and B--Bd)
   coordinate (PbonH)
  (intersection of Bd--O and Pbon--Pm)
   coordinate (Bdm)
  (intersection of Bd--O and Pb--Pm)
    coordinate (Bdh);
 \draw[shorten <=2pt,shorten >=2pt]
(PbonM)--(Ph)--(Pm)--(Pb)—
(Pbon)--(PbonH) (Bdh)--(Bd);
 \draw[shorten >=-1pt] (O)--(Bdm);
 \draw[dashed,dash pattern=on 2pt off 1.5pt]
  (PbonM)--(PbonH) (Bdm)--(Bdh);
\end{scope}
\end{tikzpicture}
```



2.4. Converting the images into png or gif file format

We sometimes need to use the images with the Microsoft Word, Microsoft Powerpoints programme. Then, the output files with pdf extension must be converted to png or gif file format.

2.4.1. Converting the images into png file format

We can convert the pdf file format to png extenstion online.

• Browse the website https://pdf2png.com

• Click the “UPLOAD FILES” button and navigate to the PDF you need to convert. Conversely, you can drag and drop your file onto the spot that says “Drop Your Files Here”.

• Once the conversion process is finished, click the “DOWNLOAD” button underneath the uploaded file. You’ll now have a ZIP file with one PNG inside that mirrors your one-page PDF.

2.4.2. Converting the images into gif file format

Website https://ezgif.com/pdf-to-gif is a simple, wide-ranging online tool for converting PDF to GIF.

You can choose to convert PDF to animated GIF, or just convert each page to individual, static GIF image. For animated GIFs, you can add fading transition between pages and select duration for each page (this option will not be available if you upload document with more than 40 pages, because it greatly increases file size).

If you upload multiple PDFs and select animated GIF option, pages from all PDF documents will be merged in one GIF.

3. Conclusion

The article introduces how to use the orthogonal projection to represent revolutions. These images are important in teaching at hight schools. After accurately calculating the critical visible points, we show the usage tikz-3d package to represent these images in LaTeX language. Moreover, details are presented on how to represent intersection of revolutions in noncanonical case. By this method, readers can represent complex images in the high school curriculum.

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